# CHURCHLANDS SENIOR HIGH SCHOOL
# MATHEMATICS SPECIALIST 3,4 TEST 1 SECTION TWO 2016 Year 12
## Calculator Section

**Name**________________________________

**Time: 35 minutes**
**Total: 35 marks**

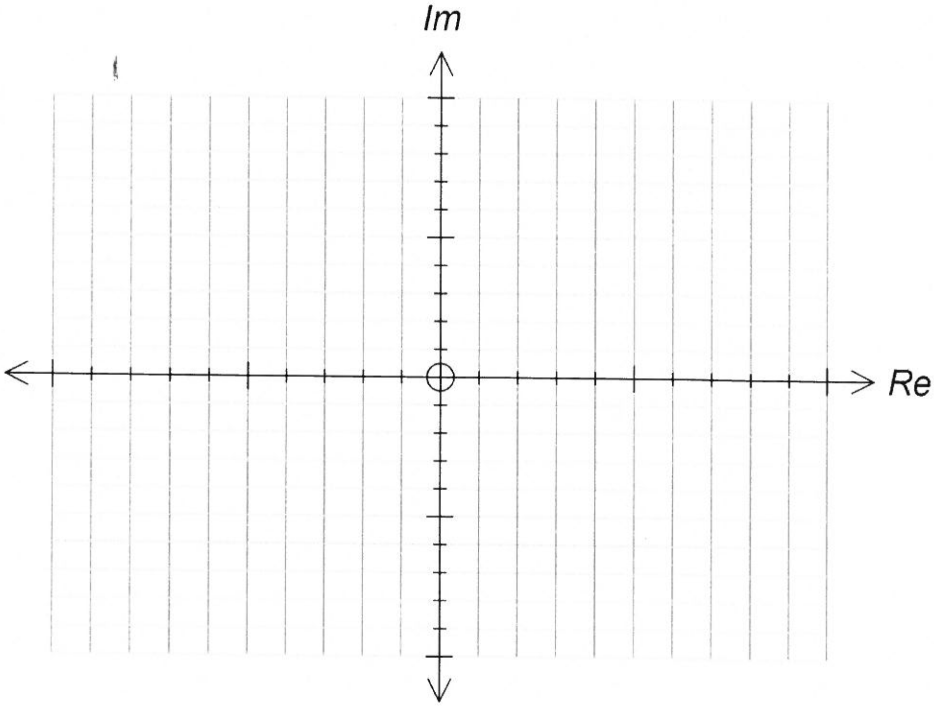
1. [1, 5, 3 marks]

(a) State the exact value of $(2 + 2\sqrt{3}i)^4$ in **Cartesian form**. (1 mark)

(b) Hence determine exact values for all the roots of $z^4 = -8 - 8\sqrt{3}i$ . (5 marks)

(c) Sketch all the roots from (b) on the Argand diagram below. Identify all the important features.

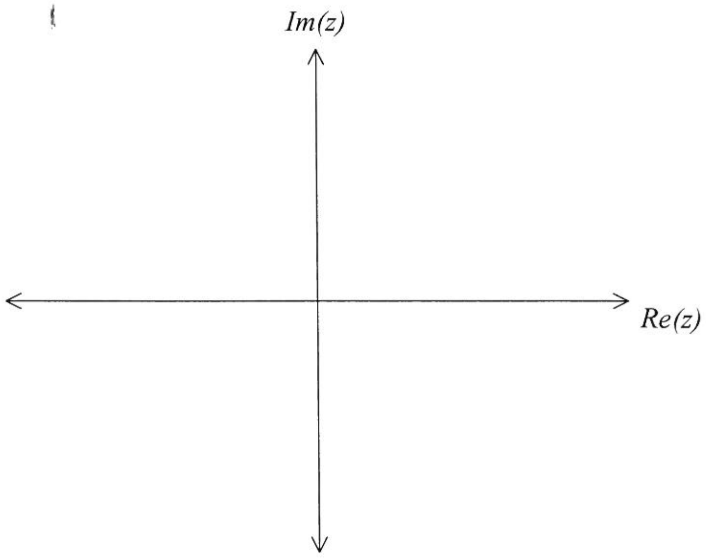
(3 marks)



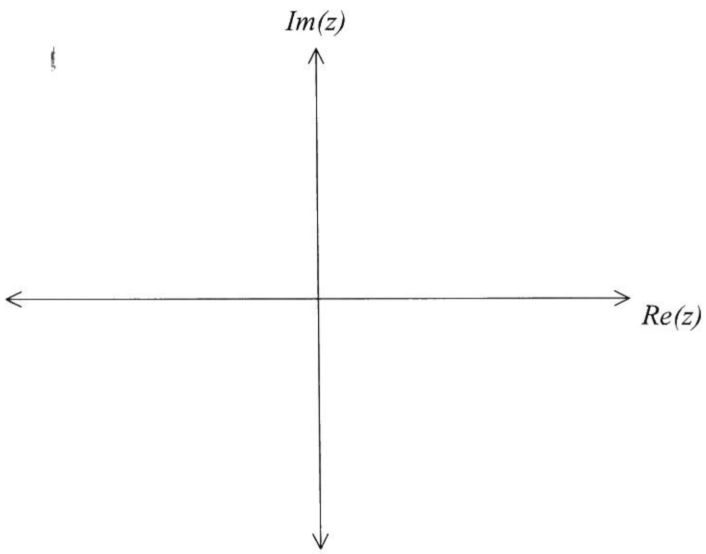
2. [2, 3, 2 marks]

Sketch the following regions in the complex plane.

(a) $-\frac{2\pi}{3} \le \arg(z) \le \frac{\pi}{3}$. (2 marks)




(b) $|z+1-2i| \ge |z+2-4i|$. (3 marks)




(c) For the region in (b) above, state the minimum value of $|z|$. (2 marks)

3. [6, 2 marks]

(a) Use de Moivre’s Theorem to solve $z^5 = -iz$

Give your answers in the form $rcis\theta$ where $r \geq 0$ and $-\pi < \theta \leq \pi$

[To obtain full marks for this question, you must show clearly the use of de Moivre's Theorem.]

(b) An exact solution to $z^5 = -iz$ is $z = \left(\frac{-\sqrt{2-\sqrt{2}}}{2}\right) + \left(\frac{-\sqrt{2+\sqrt{2}}}{2}\right) i$

Given that $cos\theta = \left(\frac{-\sqrt{2-\sqrt{2}}}{2}\right)$, use your answer in (a) and the above solution to $z$ to show that $\theta = -\frac{5\pi}{8}$. Explain clearly how you arrived at your answer.

4 [4, 3, 3 marks]

(a) Express each of the following in polar form such that $r \geq 1$ and $0 \leq \theta \leq 2\pi$. (4)

(i) $(1 - i)^5$

(ii) $(-\sqrt{3} - i)^4$

(iii) $(-1 + i\sqrt{3})$

(iv) $(-2 + 2i)^3$

(b) **Hence,** simplify $\dfrac{(1-i)^5(-\sqrt{3}-i)^4}{(-1+i\sqrt{3})(-2+2i)^3}$ giving your answer in Cartesian form. Your working steps must show clearly how you multiply and divide complex numbers expressed in polar form. (3)

(c) The complex number $z$ is given such that $\bar{z} = \frac{-1-i}{1-\sqrt{3}i}$.

Find $z$ *and*, $\frac{|z|^2}{\bar{z}}$ and hence state a relationship between them. (3)

**CHURCHLANDS SENIOR HIGH SCHOOL**
**MATHEMATICS SPECIALIST 1, 2 TEST 1 SECTION TWO 2016 Year 12**
**Calculator Section**

**Name**______________________________

**Time: 35 minutes**
**Total: ~~34~~ 33 marks**

1. [1, 5, 3 marks]

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(a) State the exact value of $(2+2\sqrt{3i})^4$ in **Cartesian form.** (1 mark)

Expand $(2+2\sqrt{3}i)^4 = -128 - 128\sqrt{3}i$

factor out $= -128(1+\sqrt{3}i)$ ✓

(b) Hence Determine exact values for all the roots of $z^4 = -8 - 8\sqrt{3}i$. (5 marks)

$= -8(1+\sqrt{3}i)$

$z^4 = -8 - 8\sqrt{3}i$

$= \frac{1}{16}(2+2\sqrt{3}i)^4$ ← recognizes $\frac{1}{16}$ ✓

→ identifies as factor from part(a) ✓

$\Rightarrow z = \frac{1}{2}(2+2\sqrt{3}i)$ → defines solution in term of part(a) ✓

$= 1+\sqrt{3}i$ → finds solution using symmetry ✓

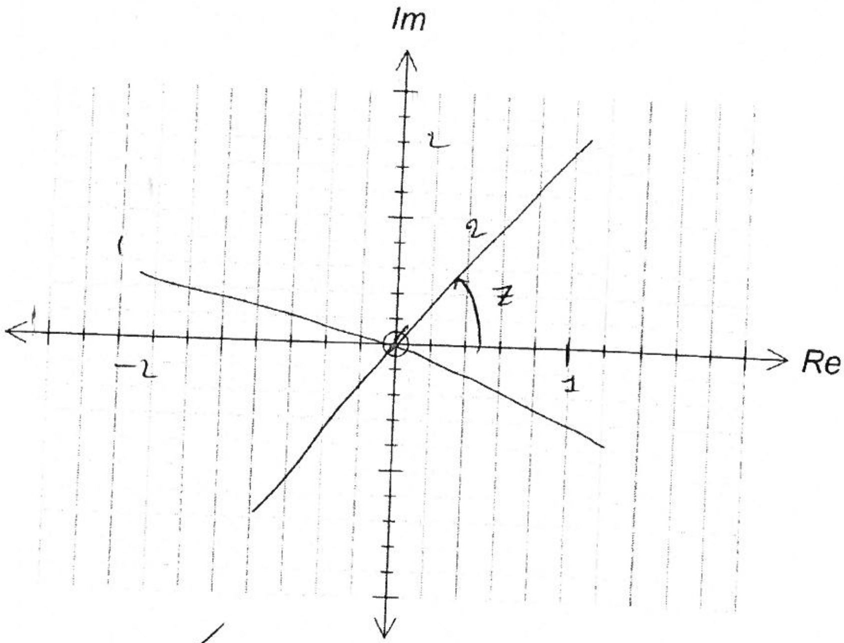
other roots equally spaced in the Argand diagram

ie rotated by $\pi/2$ from 1st root

$z = -\sqrt{3}+i$ , $-1-\sqrt{3}i$ $\sqrt{3}-i$

(c) Sketch all the roots from (b) on the Argand diagram below. Identify all the important features.

(3 marks)




Symmetric ✓ each making 90° with neighbouring roots ✓

Correct scale. ✓

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2. [2, 3, 2 marks]
Sketch the following regions in the complex plane.

$4y = 2x + 5$
$y = \frac{1}{2}x + \frac{5}{4}$
$2x - 4y = 5$

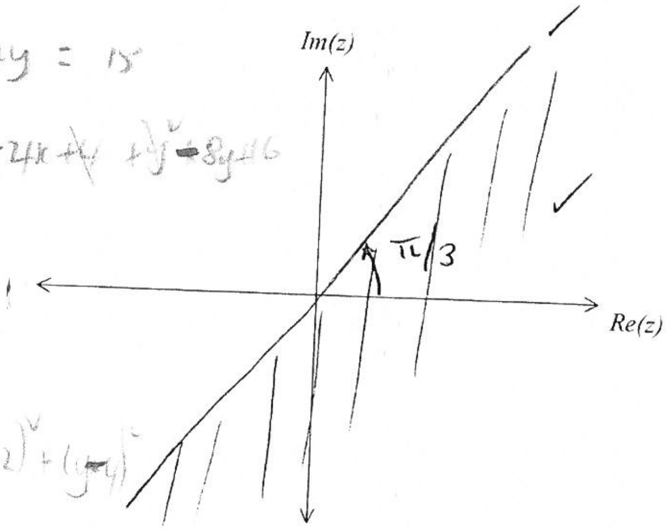
(a) $-\frac{2\pi}{3} \le \arg(z) \le \frac{\pi}{3}$. (2 marks)

$-2x + 4y = 15$

$x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 + 4x + 4 + y^2 - 8y + 16$
$6x - 15 - 12y = 0$
$2x - 5 - 4y = 0$

$(x + yi) - (-1 + 2i)$

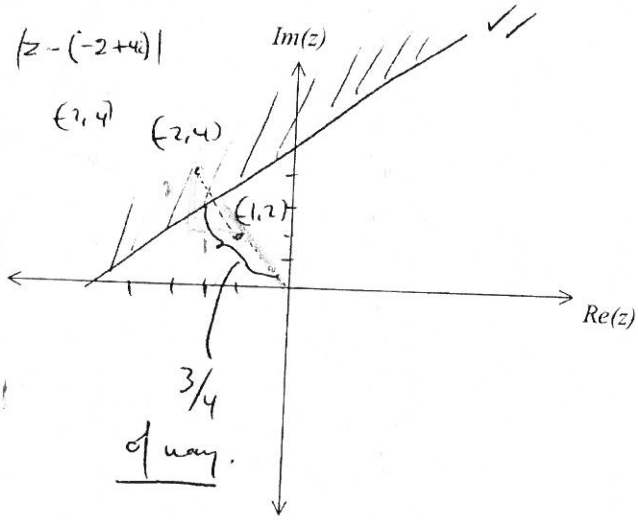
$(x+1)^2 + (y-2)^2 = (x+2)^2 + (y-4)^2$




(b) $|z + 1 - 2i| \ge |z + 2 - 4i|$. (3 marks)

$|z - (-1+2i)|$ $|z - (-2+4i)|$

$(-1, 2)$ $(-2, 4)$




(c) For the region in (b) above, state the minimum value of $|z|$. (2 marks)

$\frac{3}{4} \times \sqrt{2^2 + 4^2} = \frac{3\sqrt{5}}{2}$

3. [6, 3 marks]

chege $z^5 + iz = 0$ (9) 25

(a) Use de Moivre's Theorem to solve $z^5 = -iz$
Give your answers in the form $r cis\theta$ where $r \geq 0$ and $-\pi < \theta \leq \pi$
[To obtain full marks for this question, you must show clearly the use of de Moivre's Theorem.]

$$z^5 = -iz$$

$$\Rightarrow z(z^4 + i) = 0 \quad \Rightarrow z = 0 \quad \text{or } z^4 = -i$$

For $z^4 = -i = 1 cis\left(-\pi/2 + 2n\pi\right)$

$$\Rightarrow z = cis\left[\left(-\pi/2 + 2n\pi\right)\right]^{1/4}$$

By deMoivres

$$z = cis\left(\frac{-\pi/2 + 2n\pi}{4}\right) = cis\left(\frac{-\pi + 4n\pi}{8}\right)$$

$$= cis\left(-\pi/8\right), cis\left(3\pi/8\right), cis\left(7\pi/8\right), cis\left(11\pi/8\right)$$

$$= \quad " \quad " \quad " \quad \left(cis -5\pi/8\right)^*$$

for $z = 0$ is same

(b) An exact solution to $z^5 = -iz$ is $z = \left(\frac{-\sqrt{2-\sqrt{2}}}{2}\right) + \left(\frac{-\sqrt{2+\sqrt{2}}}{2}\right) i$

Given that $cos\theta = \left(\frac{-\sqrt{2-\sqrt{2}}}{2}\right)$, use your answer in (a) and the above solution to $z$ to show that $\theta = -\frac{5\pi}{8}$. Explain clearly how you arrived at your answer.

$z$ is located in the $3^{rd}$ quadrant as both Re & Im components are -ve.

now $cis\left(-\frac{5\pi}{8}\right) = cos\left(-\frac{5\pi}{8}\right) + i\,sin\left(-\frac{5\pi}{8}\right)$

$\therefore \quad cos\left(-\frac{5\pi}{8}\right) = \frac{-\sqrt{2-\sqrt{2}}}{2} \quad \therefore \theta = -5\pi/8$

4 [2, 3, 3 marks]

(a) Express each of the following in polar form such that $r \geq 1$ and $0 \leq \theta \leq 2\pi$. (2) 33

(i) $(1-i)^5 = 4\sqrt{2}\ \text{cis}\left(3\pi/4\right)$

(ii) $(-\sqrt{3}-i)^4 = 16\ \text{cis}\left(\frac{2\pi}{3}\right)$

(iii) $(-1+i\sqrt{3}) = 2\ \text{cis}\left(2\pi/3\right)$

(iv) $(-2+2i)^3 = 16\sqrt{2}\ \text{cis}\left(\pi/4\right)$

all values of $\theta$ correct
−1 if $\theta$ not in specified range.

(b) **Hence,** simplify $\frac{(1-i)^5(-\sqrt{3}-i)^4}{(-1+i\sqrt{3})(-2+2i)^3}$ giving your answer in Cartesian form.
Your working steps must show clearly how you multiply and divide complex numbers expressed in polar form.
(3)

$$\frac{4\sqrt{2}\ \text{cis}(3\pi/4)\ 16\ \text{cis}\ 2\pi/3}{2\ \text{cis}(2\pi/3)\ 16\sqrt{2}\ \text{cis}\ \pi/4} = \frac{4\sqrt{2}.16\ \text{cis}\left(\frac{9\pi+8\pi}{12}\right)}{\ldots}$$

$$= 2\ \text{cis}\left(\frac{17\pi}{12} - \frac{11\pi}{12}\right)$$

$$= 2\ \text{cis}\ \pi/2$$

$$= 2i$$

(c) The complex number $z$ is given such that $\bar{z} = \frac{-1-i}{1-\sqrt{3}i}$.

Find $z$ , $\frac{|z|^2}{\bar{z}}$ and hence state a relationship between (3)

$\therefore |z| = \frac{\sqrt{2}}{2}$

$z = \frac{\sqrt{3}-1}{4} + i\left(\frac{\sqrt{3}+1}{4}\right)$

clan pad!

$\frac{|z|^2}{\bar{z}} = \frac{\sqrt{3}-1}{4} + i\,\frac{\sqrt{3}+1}{4}$

$\therefore \frac{|z|^2}{\bar{z}} = z$